Patent Licensing by a Standard Auction in Presence of Network Externality

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Abstract

In this paper we develop a model of licensing, a new product technology of a network good. The new technology improves the quality of the network good and is protected by a patent. In such a context we find that a *standard* auction mechanism can *efficiently* allot the new technology both under *complete* and *incomplete information*. We consider two possible downstream market structures – one where the firms *simultaneously* choose prices and the other where they *sequentially* choose prices. We find that the equilibrium bids and expected auction revenue are more when the firms move simultaneously in the posterior price game. We also find that the equilibrium bids and expected auction revenue decreases as the network effect becomes stronger. (118 words)

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1. Introduction

The issue of *technology adoption and compatibility* in presence of *network externality* has now become a classic issue in the literature of industrial organization theory. Katz and Shapiro (1986 a, b) first identified the problem in adopting a new technology in the market for a network good. These papers, however, assumes that the utility of a consumer depends only on the network size of the product. De Bijl and Goyal (1995) examines a duopoly where the firms have to decide simultaneously on product innovation and the compatibility of the successor technologies with the established industry standard. They consider that the consumer's utility not only depends on network size but also on the technology on which the product is based. Shy (1996) analyze the technology adoption decision of a monopolist and identifies that whether a new technology will be adopted or not depends on whether the consumers perceive network size and technological quality as substitutes or as complements. This paper shows that a new technology is adopted only when technological quality and network sizes are substitutes in the consumer's perspective.

None of the above papers discuss how a new technology is allotted to the firms. However, extensive literature can be found on allocation of a cost reducing innovation to the firms that compete in the product market. A general mechanism is to allot the firms a license to use an innovation that is protected by a patent. The literature on patent races, patent licensing and value of patents is surveyed in Tirole (1988) and Kamien (1992). It is a well-known feature that the value of the patent and the efficiency of the licensing procedure depend on the downstream market structure[1].

By and large the existing literature on patent licensing assumes complete information regarding all relevant parameters. Only very recently Moldovanu and Sela (2003) developed a model of patent licensing under incomplete information. Moldovanu and Sela (2003) show that under incomplete information a *standard*[2] auction mechanism can not *efficiently*[3] allot the license to use a patented cost reducing innovation among Bertrand competitors. They also designed a *non-standard* auction mechanism, where the firm with the lowest bid wins the license and pays the second lowest bid, in order to generate efficiency.

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[1] Arrow (1962) first defined the value of a patent and pointed out that the value depends on the downstream market structure.

[2] In a standard auction the highest bidder wins. Here we will consider a first price sealed bid auction and a second price sealed bid auction. Since we have only two bidders, the second price sealed bid auction is equivalent to an English auction (i.e., open ascending auction) and the first price sealed bid auction is equivalent to a Dutch auction (i.e., descending auction).

[3] An auction is said to be efficient if it can allot the good to the bidder having the highest valuation.

The present paper attempts to complement the existing literature on technology adoption in presence of network externality by linking it to the literature on patent licensing. In this paper we develop a model of patent licensing of a technology that improves the product quality of a network good, to firms that compete in price. We find that a standard auction mechanism can efficiently allot the new technology both under complete information and incomplete information.

This result is very different from the fundamental result of Moldovanu and Sela (2003). But this is not surprising, because our model is set-up in a context that is very different from the context in which Moldovanu and Sela set up their model. In Moldovanu and Sela (2003), homogenous product Bertrand competitors bid to acquire a cost-reducing innovation. In contrast, we set up a model where the firms bid for an innovation, which improves the quality of the product. The opposing results are artifacts of this difference in set-ups. A more precise intuition is provided in section 5, after we introduce our basic result in propositions 3 and 4.

The paper is organized as follows. In section 2 we introduce the basic model. In section 3 we analyze the downstream price game where the firms move simultaneously. In section 4 and 5 we show that a standard auction can efficiently allot the license to use the new technology, respectively under complete and incomplete information. In section 6 we extend our model by modifying the timing of the downstream price game. Finally, in section 7, we conclude.

2. The Model

Let us consider a market for a network good where two *risk neutral* firms, 1 and 2, compete in prices. Initially, in period 0, both the firms base their product on the same technology, T_0, and hence the competing products are perfectly compatible with each other. Here we assume that the product quality is determined by the product technology. Therefore, when the firms base their products on the same technology, the products are homogenous. In the beginning of period 1, an innovator appears with a new technology T_1 that embodies a better quality vis-à-vis T_0, protected by a patent, which will be licensed to either of the firms by a standard auction mechanism. So in period 1 first the firms bid for the new technology and after the license to use the new technology is allotted to the highest bidder, the firms compete in prices.

In any period there exists a continuum of consumers distributed uniformly in the interval [0, 1], according to their type θ. The type of a consumer reflects her willingness to pay for the new technology. Each consumer has a perfectly inelastic demand, i.e., they either buy one unit of the commodity or they do not buy it. Therefore, the potential market size in any period is unity. The indirect utility to a period t consumer, of type θ, purchasing from firm i, is given by,

$$V_{it} = \alpha N_{it} - P_{it} \qquad \text{if the firm i base its product on the technology } T_0.$$
$$= \alpha N_{it} + \theta - P_{it} \qquad \text{if the firm i base its product on the technology } T_1. \qquad (1)$$

N_{it} is the network size[4] of the firm i in period t, α is the *coefficient of network externality*, which is a measure of the network effect and P_{it} is the price charged by firm i in period t. Larger the α, stronger is the network effect. Since we are considering positive externality, α is strictly positive. If a consumer purchases a product based on the technology T_0, then she derives utility only due to the network effect. If, however, she purchase the product based on the technology T_1, she derives an additional utility according to her type θ. In a sense we are normalizing the product differential to 1 and hence the additional utility that a consumer gets from purchasing the product based on the new technology simply depends on her type.

Since, in period 0, both the firms base their product on the technology T_0, each consumer get the same indirect utility from purchasing the good. Also since the products are perfectly compatible, the network size is same as total sales. Therefore, in equilibrium, either all the consumers purchase the good or none of them purchase it. If all the consumers purchase the good the network size is unity. In equilibrium all the consumers purchase the good if and only if each of them get at least a non-negative consumers' surplus. Therefore, all the consumers purchase the good in equilibrium iff, $P_{i0} \leq \alpha$, for all i = 1, 2. Since the products are perfectly homogenous, in equilibrium, all the consumers buy from the firm charging the lower price. If both the firms charge the same price the market is equally shared. So, price competition will pull down the price to marginal cost and the firms equally share the market. For simplicity we assume *zero cost of production*. Therefore, in period 0, the price comes down to zero and, in equilibrium, all the consumers purchase the good, i.e., $N_{i0} = 1$ for all i = 1, 2.

The period 0 consumers are alive in period 1 and hence contribute to the network size of the old technology in period 1. However the period 0 consumers do not get any utility from their consumption in period 1[5].

In period 1, the innovator appears with the new technology T_1 that embodies a better quality, protected by a patent, which she will license to either of the firms through a standard auction. The technology T_1 is perfectly incompatible with the existing technology T_0. However, a product

[4] N_{it} is actually the expected network size and we assume that the expectations are fulfilled in equilibrium.

[5] This stylization allows us to avoid the issue of Coasian dynamics (Coase 1972).

based on T_1 can be made partially compatible with the product based on T_0. The degree to which a product based on T_1 will be partially compatible to the products based on T_0 depends on the ability to achieve partial compatibility on part of the firm that acquires the new technology. Suppose the firm i (i = 1, 2) get the technology T_1 and let γ_i be degree of partial compatibility that can be achieved by firm i, where $\gamma_i \in [0, 1]$. At $\gamma_i = 0$, the products are perfectly incompatible and at $\gamma_i = 1$ they are perfectly compatible. We define partial compatibility in the following manner. Let q_{i1} is the sales of firm i (i = 1, 2) in period 1, as a proportion of potential market. Note that since all the period 0 consumers purchased the good based on T_0, the technology T_0 has an installed base of size 1. In period 1 firm i base its product on T_1 but firm j ($j \neq i$) base its product on T_0. Therefore, the mass of consumers using product based on T_0 is $(1 + q_{j1})$ and the mass of consumers using a product based on T_1 is q_{i1}. Given the degree of partial compatibility γ_i, the effective network size[6] of firm i is given by,

$$N_{i1} = q_{i1} + \gamma_i(1 + q_{j1}) \quad (2)$$

and the effective network size of firm j is given by,

$$N_{j1} = \gamma_i \, q_{i1} + (1 + q_{j1}). \quad (3)$$

The timing of the game in period 1 is as follows. In stage 1 the firms bid for the new technology. In stage 2, after one of the firms acquires the license to use the new technology, they compete in prices. We will consider the case where the stage 1 game is conducted under complete information as well as the case where it is conducted under incomplete information. When the stage 1 game is conducted under complete information the value of γ_i is known to both i and j, i = 1, 2, $j \neq i$. When it is conducted under incomplete information γ_i is known to i but not to j. γ_i can be thought as the type of firm i. Under incomplete information, the firms do not know the type of the rival. At the end of stage 1, the value of γ_i (i = 1, 2) are revealed to all even when the stage 1 game is conducted under incomplete information. So irrespective of the fact that the stage 1 game had been conducted under complete information or not, the posterior price game is conducted under complete information. This specification allows us to avoid the issue of signaling through bids in order to influence competitor's beliefs in the downstream interaction game[7]. We solve this game by the method of backward induction. Before we move on to the following section where we analyze the downstream price game, we make two assumptions.

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[6] Shy (1996) uses this definition of partial compatibility. Chou and Shy (1993) also defines partial compatibility in terms of percentage of software specifically written for a particular operating system that can be run on another operating system.

[7] It is well known that such signaling may cause inefficiencies. See Das Varma (2003), Goeree (2000).

Assumption 1: γ_i (i = 1, 2) are independently and identically distributed and follows uniform distribution in the interval [1/2, 1].

Assumption 2: $\alpha \in (1/4, 1)$.

These two assumptions ensure the existence of interior solution in the downstream price game[8].

3. The downstream price game

Let the firm i win the new technology T_1, in a standard auction in stage 1. Since i can be either 1 or 2, this is perfectly general. In period 1 there exist a continuum of consumers distributed uniformly, according to their type θ, in the interval [0, 1]. It is apparent from (1) that if a consumer of type θ^* purchase the new technology product from i then all consumers of type $\theta \geq \theta^*$ purchase it. By continuity of θ, there exist a consumer of type θ^* such that all consumers of type $\theta \geq \theta^*$ purchase the new technology product from i and the consumers of type $\theta < \theta^*$ either purchase the network good from j or they do not buy it. Since the consumer's indirect utility from purchasing the old technology product from j is independent of their type θ, in equilibrium, either all the consumers of type $\theta < \theta^*$ purchase the good from j or none of them buy it. If all the period 1 consumers of type $\theta < \theta^*$ purchase from j, then the full market is covered and the sales of i and j in period 1 are respectively given by, $q_{i1} = (1 - \theta^*)$ and $q_{j1} = \theta^*$. Using our definition of partial compatibility, the effective network sizes of firm i and j, in period 1, are respectively given by,

$$N_{i1} = (1 - \theta^*) + \gamma_i(1 + \theta^*) \quad (4)$$

and,

$$N_{j1} = \gamma_i (1 - \theta^*) + (1 + \theta^*), \quad (5)$$

where γ_i is the degree of partial compatibility achieved by the firm i who got the new technology T_1. In this case, the indirect utility of a period 1 consumer of type $\theta \geq \theta^*$, purchasing from firm i, is given by,

$$V_{i1} = \alpha [(1 - \theta^*) + \gamma_i(1 + \theta^*)] + \theta - P_{i1} \quad (6)$$

and the indirect utility of a consumer of type $\theta < \theta^*$, purchasing from j, is given by,

$$V_{j1} = \alpha [\gamma_i (1 - \theta^*) + (1 + \theta^*)] - P_{j1} \quad (7)$$

Clearly, a necessary condition for the consumers of type $\theta < \theta^*$ to purchase the network good based on the old technology is,

$$P_{j1} \leq \alpha [\gamma_i (1 - \theta^*) + (1 + \theta^*)] \quad (8)$$

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[8] When the firms move simultaneously in the downstream price game $\alpha \in (1/6, 1)$ ensures existence of interior solution. However when the firms move sequentially in the downstream price game, the existence of interior solution is ensured when $\alpha \in (1/4, 1)$. Therefore, $\alpha \in (1/4, 1)$ is a sufficient condition for existence of interior solution in both forms of downstream market structure.

The consumer of type θ* is indifferent between purchasing from i and j. By definition,

$$\theta^* = \frac{P_{i1} - P_{j1}}{1 - 2\alpha(1-\gamma_i)} \tag{9}$$

Our assumptions that $\gamma_i \in [1/2, 1]$ and $\alpha \in (1/4, 1)$ ensure that $1 - 2\alpha(1 - \gamma_i) > 0$.

Substituting the value of θ*, from (9), in (8), we get the necessary condition for the consumers of type $\theta < \theta^*$ to purchase the network good based on the old technology as,

$$P_{j1} \leq \alpha\left[\frac{(1+\gamma_i)\{1-2\alpha(1-\gamma_i)\} + (1-\gamma_i)P_{i1}}{1-\alpha(1-\gamma_i)}\right] \tag{10}$$

Now we can write the demand functions for firm i and firm j, in period 1, as,

$$q_{i1} = \frac{1-2\alpha(1-\gamma_i) - P_{i1} + P_{j1}}{1-2\alpha(1-\gamma_i)} \qquad \text{if, } P_{i1} < 1 - 2\alpha(1-\gamma_i) + P_{j1}$$

$$= 0 \qquad \text{otherwise.} \tag{11}$$

and, $$q_{j1} = \frac{P_{i1} - P_{j1}}{1-2\alpha(1-\gamma_i)} \qquad \text{if, } P_{j1} < P_{i1} \text{ and,}$$

$$P_{j1} \leq \alpha\left[\frac{(1+\gamma_i)\{1-2\alpha(1-\gamma_i)\} + (1-\gamma_i)P_{i1}}{1-\alpha(1-\gamma_i)}\right]$$

$$= 0 \qquad \text{otherwise.} \tag{12}$$

Therefore, the profit functions of i and j, respectively, are

$$\Pi_{i1} = \frac{1-2\alpha(1-\gamma_i) - P_{i1} + P_{j1}}{1-2\alpha(1-\gamma_i)} P_{i1} - \beta \qquad \text{if, } P_{i1} < 1 - 2\alpha(1-\gamma_i) + P_{j1}$$

$$= -\beta \qquad \text{otherwise.} \tag{13}$$

and, $$\Pi_{j1} = \frac{P_{i1} - P_{j1}}{1-2\alpha(1-\gamma_i)} P_{j1} \qquad \text{if, } P_{j1} < P_{i1} \text{ and,}$$

$$P_{j1} \leq \alpha\left[\frac{(1+\gamma_i)\{1-2\alpha(1-\gamma_i)\} + (1-\gamma_i)P_{i1}}{1-\alpha(1-\gamma_i)}\right]$$

$$= 0 \qquad \text{otherwise.} \tag{14}$$

Where, β is the fixed cost incurred by firm i to buy the new technology T_1, through a standard auction, in stage 1.

From the first order conditions for profit maximization we get the price reaction functions as,

$$P_{i1} = \frac{1 - 2\alpha(1-\gamma_i) + P_{j1}}{2} \tag{15}$$

and, $$P_{j1} = \frac{P_{i1}}{2} \quad \text{iff, } P_{j1} \le \alpha\left[\frac{(1+\gamma_i)\{1-2\alpha(1-\gamma_i)\} + (1-\gamma_i)P_{i1}}{1-\alpha(1-\gamma_i)}\right] \tag{16}$$

Solving the reaction functions we get the equilibrium prices as,

$$P_{i1} = \frac{2\{1-2\alpha(1-\gamma_i)\}}{3} \tag{17}$$

$$P_{j1} = \frac{1-2\alpha(1-\gamma_i)}{3} \tag{18}$$

Our assumptions that $\gamma_i \in [1/2, 1]$ and $\alpha \in (1/4, 1)$ ensure that the condition $P_{j1} \le \alpha\left[\frac{(1+\gamma_i)\{1-2\alpha(1-\gamma_i)\} + (1-\gamma_i)P_{i1}}{1-\alpha(1-\gamma_i)}\right]$ is satisfied.

Substituting the equilibrium prices in (11) and (12) we get the equilibrium quantities as $q_{i1} = 2/3$ and $q_{j1} = 1/3$, and the equilibrium profits are,

$$\Pi_{i1} = \frac{4\{1-2\alpha(1-\gamma_i)\}}{9} - \beta \tag{19}$$

$$\Pi_{j1} = \frac{1-2\alpha(1-\gamma_i)}{9} \tag{20}$$

From the above discussion we arrive at the following proposition.

Proposition 1: There exists a unique interior solution in the downstream price game when the firms move simultaneously. The firm with the new technology, firm i, earns a gross profit of $4/9[1 - 2\alpha(1 - \gamma_i)]$ and the firm with the old technology, firm j, earns $1/9[1 - 2\alpha(1 - \gamma_i)]$.

In the following two sections we analyze the stage 1 game, where the competing firms bid for the license to use the new technology, T_1, in a standard auction. Before proceeding to the next section we define the value of the new technology to the firms. In a classical paper, Arrow (1962) defined the *intrinsic value* of a patent, on a cost reducing innovation, as the revenue that the innovator could achieve by licensing the innovation to producing firms. Following Arrow (1962), Moldovanu and Sela (2003) defined the *intrinsic value* of a cost reducing innovation as the difference between the profit a firm makes, in case it acquires the innovation, and the profit, in case it does not. Here we are not considering a cost reducing innovation, but a product improving innovation. But, since the same intuition applies, we adopt the definition of Moldovanu and Sela (2003), i.e., the *intrinsic value* of the new technology, T_1, as the difference between the profit a firm makes, in case it acquires the new technology, and the profit, in case it does not. Therefore, using proposition 1, we formally define the *intrinsic value* of the new technology as follows:

Definition 1: When the firms move simultaneously in the downstream price game, the value of the new technology to the competing firms is given by,

$$v_i = \frac{4}{9}[1-2\alpha(1-\gamma_i)] - \frac{1}{9}[1-2\alpha(1-\gamma_j)], \qquad i = 1, 2; j \neq i \tag{21}$$

4. Firms bid for the new technology under complete information

It is apparent from (21) that the ex-ante value of the new technology to firm i depends on both γ_i and γ_j and hence the auction is not a *private values* one but an *interdependent values* auction. One must note that in our model the interdependence arises from the fact that there exists network externality and the network size is determined by the degree of partial compatibility achieved by the firm acquiring the new technology.

The value of the new technology to firm i (i = 1, 2) depends also on α, the coefficient of network externality. The magnitude of the coefficient of network externality is a common knowledge. Each firm knows its own ability in terms of achieving partial compatibility, i.e., firm i knows γ_i. In this section we assume that γ_j is also known to firm i. So, the standard auction takes place under *complete information*.

Proposition 2: When the license to use the new technology is allotted by a standard auction, the allocation is efficient. The auction revenue is, $v_L = \min\{v_i, v_j\}$.

Proof: Without loss of generality, let us assume $\gamma_1 > \gamma_2$. Therefore, from (21), $v_1 > v_2$. Consequently, firm 1 wins the license to use the new technology by bidding v_1 in a second price sealed bid auction, and by bidding v_2 in a first price sealed bid auction. Therefore, the firm with a higher valuation obtains the license to use the new technology, and hence the allocation is efficient. Both in the first price as well as the second price sealed bid auction, the winner (firm 1) pays v_2 and the revenue to the innovator is $v_2 = \frac{4}{9}[1-2\alpha(1-\gamma_2)] - \frac{1}{9}[1-2\alpha(1-\gamma_1)]$. □

From proposition 2 we conclude that a standard auction can efficiently allot the license to use the new technology when the firms know each other's ability in terms of achieving partial compatibility.

5. Firms bid for the new technology under incomplete information

In this section we allow for ex-ante incomplete information regarding the firms' ability to achieve partial compatibility. Firm i (i = 1, 2) knows the exact value of γ_i but not of γ_j ($j \neq i$). However, it is common knowledge that γ_i (i = 1, 2) independently follows uniform distribution in the interval [1/2, 1].

We show that a standard auction can efficiently allot the license to use the new technology even under incomplete information. This result is not surprising given that the firms are ex-ante *symmetric*. The term *symmetric* refers to the fact that the firms have symmetric value functions. Generally, if the bidders are ex-ante symmetric and a standard auction is efficient under complete information, it is efficient under incomplete information as well[9]. We will also show that the expected revenue is same in case of first price and second price sealed bid auctions. Revenue equivalence is well known in case of private value auctions. Our model provides an interdependent values auction and yet the revenue equivalence continues to hold.

Proposition 3: In the second price sealed bid auction, each bidder adopting $b_i^{II}(\gamma_i) = \frac{1}{3}[1 - 2\alpha(1-\gamma_i)]$, (i = 1, 2) constitute a *symmetric* Bayesian Nash equilibrium.

[Proof given in the appendix.]

From proposition 3 we arrive at the following conclusions. Firstly, the second price sealed bid auction is efficient, i.e., it allots the license to use the new technology to the firm with the highest valuation[10]. From 21 we know that the value of the new technology to firm i (i = 1, 2) is monotonically increasing in γ_i, for all $\alpha > 0$. The equilibrium bid in the second price sealed bid auction is also monotonically increasing in γ_i, for all $\alpha > 0$. Therefore, the firm with the larger

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[9] See Myerson (1981) and Milgrom and Weber (1982).

[10] In auction theory it is a well-known fact that a standard interdependent value auction is efficient if the *single crossing condition* is satisfied. This condition, in the context of our model, requires $\frac{\partial v_i}{\partial \gamma_i} \geq \frac{\partial v_j}{\partial \gamma_i}$, which is satisfied. The single crossing condition ensures that the ex-post values of different bidders are ordered in the same way as their γ_i's. For a detailed discussion see Krishna (2002).

coefficient of partial compatibility has the highest valuation as well as the highest bid and wins the auction. Secondly, we observe that the equilibrium bids are decreasing in α. As the network effect becomes stronger the firms becomes reluctant to switch to a new technology[11].

Now let us consider the case when the standard auction is a first price sealed bid auction.

Proposition 4: In the first price sealed bid auction, each bidder adopting $b_i^I(\gamma_i) = \frac{1}{6}[2 - \alpha - 2\alpha(1-\gamma_i)]$, (i = 1, 2) constitute a *symmetric* Bayesian Nash equilibrium.

[Proof given in the appendix.]

From proposition 4 it is apparent that the equilibrium bids, in case of a first price auction, are increasing in the firm's coefficient of partial compatibility and hence, the firm with the largest coefficient (and hence the highest valuation for the new technology) gets the license to use the new technology. So, the first price auction is also efficient. The equilibrium bids in this case are also decreasing in the coefficient of network externality.

It is not surprising that the results derived in proposition 3 and 4 are opposite to the fundamental result of Moldovanu and Sela (2003). In Moldovanu and Sela (2003), homogenous product Bertrand competitors bid to acquire a cost-reducing innovation. In their set-up, the firm acquiring the innovation can capture the entire market by setting price just below the cost of the other firm. Since the loser goes out of business, the value of the innovation is same as the profit of the winner. In this set-up, if the innovation is allotted by a second price auction and the firms do not know each other's cost, the (symmetric) equilibrium bid function is monotonically increasing in cost. This happens because if a low cost firm outbids a low cost firm, it earns a low profit, conditional on winning. However, a high cost firm expects to earn a high profit. Therefore, the marginal willingness to pay is higher for a high cost firm. This phenomenon leads to inefficient allocation by a standard auction. In contrast, in our model the firms bid for a new technology that improves the quality of a network good. Whoever gets the new technology can charge a higher price and get a larger share of the market, but both firms earn positive profits in equilibrium. In presence of network externality the profits of both firms are increasing in degree of partial compatibility. The onus of achieving partial compatibility lies on the firm acquiring the new technology. Even if the firms do not know each other's γ, each firm knows the magnitude of its

[11] In the literature of network externality this phenomenon is known as *excess inertia*. See Katz and Shapiro (1985, 1986 a, b), Farrell and Saloner (1985, 1986a, b).

profit, conditional on winning. Therefore, the firm with a higher γ has a larger willingness to pay for the new technology, which in turn leads to efficient allocation by a standard auction.[12]

Proposition 5: The expected revenue in case of second price as well as first price sealed bid auction is $\frac{1}{3}\left(1-\frac{2\alpha}{3}\right)$.

[Proof given in the appendix.]

Proposition 5 shows that the expected revenue to the seller is the same from the second price auction and the first price auction. Revenue equivalence holds despite the fact that our model is an interdependent value one. One must note that the expected revenue is decreasing in α. As the network effect becomes stronger the firms become reluctant to adopt the new technology due to *excess inertia*, which in turn reduces the expected auction revenue of the innovator.

6. An extension – The firms move sequentially in the downstream price game: Endogenous leadership

In this section we modify the timing of game by allowing the firms to move sequentially in the downstream price game and the *leader* is determined endogenously. To do this we focus on a continuous version of the discrete 'timing game' introduced by Robson (1990). For the continuous version one can refer to Bergman (1997) and Canoy and Van Cayseele (1996). From the results of these papers it follows that firm i will be the *endogenous price leader* whenever, $(\Pi_i^L - \Pi_i^F) > (\Pi_j^L - \Pi_j^F)$, where Π_i^L and Π_i^F are the profits of firm i, when it is the *leader* and when it is the *follower*, respectively (i = 1, 2, j ≠ i)[13]. Apart from this modification, all other basic assumptions of our model remain unchanged. Here also we solve the game by backward induction.

First let us solve the downstream price game. As in section 3, let the firm i (i = 1, 2) win the new technology T_1, in the auction in stage 1. The demand functions of firm i and j (j ≠ i) are respectively given in (11) and (12) and the profit functions are given in (13) and (14).

Firm i is the leader:

Considering firm i as the leader and solving the pricing sub-game by backward induction we get the equilibrium prices as,

[12] I thank the referee for providing the clue to this intuitive explanation.

[13] Dastidar and Furth (2003) also adopted this approach to analyze price leadership.

$$P^{L}_{i1} = 1 - 2\alpha(1-\gamma_i) \quad (22)$$

$$P^{F}_{j1} = \frac{1 - 2\alpha(1-\gamma_i)}{2} \quad (23)$$

Since, $\alpha \in (1/4, 1)$ and $\gamma_i \in [1/2, 1]$, the condition for the existence of interior solution [condition (10)] is satisfied.

Substituting the equilibrium prices in the demand functions we get the equilibrium quantities as $q_{i1} = 1/2$ and $q_{j1} = 1/2$, and the equilibrium profits are,

$$\Pi^{L}_{i1} = \frac{\{1 - 2\alpha(1-\gamma_i)\}}{2} - \beta \quad (24)$$

$$\Pi^{F}_{j1} = \frac{1 - 2\alpha(1-\gamma_i)}{4} \quad (25)$$

Firm j is the leader:

Now let firm j be the leader. Solving the pricing sub-game by backward induction we get the equilibrium prices as,

$$P^{F}_{i1} = \frac{3\{1 - 2\alpha(1-\gamma_i)\}}{4} \quad (26)$$

$$P^{L}_{j1} = \frac{1 - 2\alpha(1-\gamma_i)}{2} \quad (27)$$

Since, $\alpha \in (1/4, 1)$ and $\gamma_i \in [1/2, 1]$, the condition for the existence of interior solution [condition (10)] is satisfied.

Substituting the equilibrium prices in the demand functions we get the equilibrium quantities as $q_{i1} = 3/4$ and $q_{j1} = 1/4$, and the equilibrium profits are,

$$\Pi^{F}_{i1} = \frac{9\{1 - 2\alpha(1-\gamma_i)\}}{16} - \beta \quad (28)$$

$$\Pi^{L}_{j1} = \frac{1 - 2\alpha(1-\gamma_i)}{8} \quad (29)$$

Both in the case where i is the leader as well as in the case where j is the leader, it is not possible for the leader to impose exit on the follower by limit pricing. Even when j is the leader it cannot impose exit on i by limit pricing, despite the fact that i incurs a fixed cost in acquiring the license to use the new technology. Once i obtains the license, the cost of obtaining it becomes *sunk*. Therefore, i will not leave the market as long as the gross profit is non-negative. As a result it is not possible for j to impose exit by limit pricing. So, when $\alpha \in (1/4, 1)$ and $\gamma_i \in [1/2, 1]$, there exist an interior solution in the sequential move price game, irrespective of who the leader is.

From (24) and (28) we get,

$$\Pi^L_{i1} - \Pi^F_{i1} = -\frac{\{1 - 2\alpha(1-\gamma_i)\}}{16} \tag{30}$$

and from (25) and (29) we get,

$$\Pi^L_{j1} - \Pi^F_{j1} = -\frac{\{1 - 2\alpha(1-\gamma_i)\}}{8} \tag{31}$$

Comparing (30) and (31) we find, $\Pi^L_{i1} - \Pi^F_{i1} > \Pi^L_{j1} - \Pi^F_{j1}$. Therefore, by our definition of *endogenous leader* firm i is the leader. Here, $(\Pi^L_{i1} - \Pi^F_{i1}) < 0$ for all i = 1, 2, i.e., there exist *second mover's advantage*. However, firm i still emerges as an endogenous leader because the advantage of being a follower over being a leader is smaller for i than it is for j.

So, when the firms move sequentially, firm i (the firm licensed to use the new technology) emerges as the leader and the equilibrium prices are given by, (22) and (23). The corresponding profits are given by (24) and (25). From the above discussion we arrive at the following proposition.

Proposition 6: There exists a unique interior solution in the downstream price game when the firms move sequentially. The firm with the new technology, firm i, emerges as the endogenous leader and, in equilibrium, earns a gross profit of $1/2[1 - 2\alpha(1 - \gamma_i)]$. The follower firm earns $1/4[1 - 2\alpha(1 - \gamma_i)]$.

Accordingly we modify our definition of *intrinsic value* of the new technology as follows: When the firms move sequentially in the downstream price game, the value of the new technology to the competing firms is given by,

$$v_i = \frac{1}{2}[1 - 2\alpha(1-\gamma_i)] - \frac{1}{4}[1 - 2\alpha(1-\gamma_j)], \qquad i = 1, 2; j \neq i \tag{32}$$

Analogous to the previous sections of this paper, we look into the stage 1 game, first under complete information and then under incomplete information.

Proposition 7: When the timing of the posterior pricing sub-game is sequential, a standard auction is efficient under *complete information* and the auction revenue is $v_L = \min\{v_1, v_2\}$.

[The proof is analogous to the proof of proposition 2.]

In both second price as well as first price auction, the firm with the highest valuation wins and hence the auction mechanisms are efficient. In both cases auction revenue is the second highest value. So, even when the firms move sequentially in the downstream price game, a standard auction can efficiently allot the license to use the new technology, in presence of complete information. Now let us consider the stage 1 game under incomplete information.

Proposition 8: In the second price sealed bid auction, under *incomplete information*, each bidder adopting $b_i^{II}(\gamma_i) = \frac{1}{4}[1 - 2\alpha(1-\gamma_i)]$, (i = 1, 2) constitute a *symmetric* Bayesian Nash equilibrium when the firms move sequentially in the downstream price game.

[Proof is analogous to the proof of proposition 3.]

The equilibrium bids in the second price sealed bid auction, under incomplete information, are monotonically increasing in γ_i and decreasing in α. The value functions given in (32) are also monotonically increasing in γ_i. Therefore, the firm with the highest valuation gets the license to use the new technology, i.e., the second price sealed bid auction is efficient even when the firms move sequentially in the posterior price game. The expected revenue is given by, $ER^{II} = \frac{1}{4}[1 - 2\alpha\{1 - E(\gamma_L)\}] = \frac{1}{4}\left(1 - \frac{2\alpha}{3}\right)$.

Proposition 9: In the first price sealed bid auction, under *incomplete information*, each bidder adopting $b_i^{I}(\gamma_i) = \frac{1}{8}[2 - \alpha - 2\alpha(1-\gamma_i)]$, (i = 1, 2) constitute a *symmetric* Bayesian Nash equilibrium when the firms move sequentially in the downstream price game.

[Proof is analogous to the proof of proposition 4.]

The equilibrium bids in the first price sealed bid auction are also monotonically increasing in γ_i and decreasing in α. Since the valuations are also monotonically increasing in γ_i, the firm with the higher valuation gets the new technology, and hence the first price sealed bid auction, under incomplete information, is efficient when the firms move sequentially in the posterior price game. The expected revenue is given by, $ER^{I} = \frac{1}{8}[2 - \alpha - 2\alpha\{1 - E(\gamma_H)\}] = \frac{1}{4}\left(1 - \frac{2\alpha}{3}\right)$, which is same as in the case of second price sealed bid auction, under incomplete information, when the firms move sequentially in the posterior price game. Therefore, the revenue equivalence holds irrespective of the timing of the downstream price game.

As in the case where the firms move simultaneously in the downstream price game, here also, the equilibrium bids and the expected revenue are decreasing in α. However, for a given α, in the interval (1/4, 1), the equilibrium bids in a second price as well as a first price auction, and hence the expected revenue, are larger when the firms move simultaneously vis-à-vis the case where they move sequentially in the posterior price game. Here, when the firms sequentially choose price in the stage 2, the firm that gets the license to use the new technology emerges as the endogenous leader, despite the existence of a *second mover's advantage*. This second mover's advantage that accrues to the firm that looses out in the auction reduces the firms' valuation for the new technology which in turn reduces the equilibrium bids and hence the expected auction revenue.

7. Conclusion:

This paper attempts to link the literature on 'technology adoption in presence of network externality' to the literature on 'patent licensing'. In this paper we show that a standard auction can efficiently allot the license to use a new technology that improves the quality of a network good. The firm with the larger coefficient of partial compatibility has a larger value for the new technology and, in equilibrium, gets the license to use the technology when the license is allotted through a standard auction mechanism. The expected auction revenue reduces, as the network effect becomes stronger. In industries where the network externality is very strong, the firms are reluctant to adopt the new technology due to the well-known phenomenon of *excess inertia*. This reduces the firms' equilibrium bids and hence the expected auction revenue. We show that the above results are independent of the timing of the downstream game. However, the firms' equilibrium bids and the expected auction revenue are more when the firms move simultaneously in the posterior price game than when they move sequentially. In the sequential move price game there exists a second mover's advantage. The firm that wins the license to use the new technology emerges as the first mover and hence cannot exercise the second mover's advantage. This phenomenon reduces the bids and hence the expected auction revenue.

A situation where the firms are engaged in an *R&D race* can be modelled within this framework. Suppose each firm bears the cost of investment in R&D, but only the firm spending the largest amount succeeds to innovate. Indeed, such a situation is isomorphic to a *first price sealed bid all pay auction* in which every agent submits and pays a bid for the item being sold, while only the highest bidder receives the item.

Appendix

Proof of proposition 3: Let $b_j^{II}(\gamma_j) = \frac{1}{3}[1-2\alpha(1-\gamma_j)]$, j ≠ i. The expected net payoff to firm i from bidding b_i is given by, $\pi_i^{II} = E\left[v_i - b_j \middle| b_i > \frac{1}{3}[1-2\alpha(1-\gamma_j)] \right]$

Or, $\pi_i^{II} = E\left[\frac{4}{9}\{1-2\alpha(1-\gamma_i)\} - \frac{1}{9}\{1-2\alpha(1-\gamma_j)\} - \frac{1}{3}\{1-2\alpha(1-\gamma_j)\} \middle| \gamma_j < (\frac{3b_i - 1}{2\alpha} + 1) \right]$

Or, $\pi_i^{II} = 2 \int_{1/2}^{(\frac{3b_i-1}{2\alpha}+1)} \left[\frac{4}{9}\{2\alpha(\gamma_i - \gamma_j)\} \right] d\gamma_j$

Or, $\pi_i^{II} = \frac{16\alpha}{9}\left(\frac{3b_i - 1}{2\alpha} + \frac{1}{2} \right)\left[\gamma_i - \frac{1}{2}\left(\frac{3b_i - 1}{2\alpha} + \frac{3}{2} \right) \right]$

Maximizing the expected net payoff with respect to b_i, we get the equilibrium bid as,

$b_i^{II}(\gamma_i) = \frac{1}{3}[1-2\alpha(1-\gamma_i)]$.

Therefore, given $b_j^{II}(\gamma_j) = \frac{1}{3}[1-2\alpha(1-\gamma_j)]$, the best response of firm i is $b_i^{II}(\gamma_i) = \frac{1}{3}[1-2\alpha(1-\gamma_i)]$. ■

Proof of proposition 4: Let, $b_j^{I}(\gamma_j) = \frac{1}{6}[2-\alpha-2\alpha(1-\gamma_j)]$ j ≠ i. The expected net payoff to firm i from bidding b_i is given by, $\pi_i^{I} = E\left[v_i - b_i \middle| b_i > \frac{1}{6}[2-\alpha-2\alpha(1-\gamma_j)] \right]$

Following steps similar to those in the proof of proposition 3, we get,

Or, $\pi_i^{I} = \left(\frac{3b_i - 1}{\alpha} + 1 \right)\left[\frac{2(1-2\alpha)}{3} + \frac{16\alpha\gamma_i}{9} - \frac{2\alpha}{9}\left(\frac{3b_i - 1}{\alpha} + 2 \right) - 2b_i \right]$

Maximizing the expected net payoff with respect to b_i, we get the equilibrium bid as,

$b_i^{I}(\gamma_i) = \frac{1}{6}[2-\alpha-2\alpha(1-\gamma_i)]$.

Therefore, given $b_j^{I}(\gamma_j) = \frac{1}{6}[2-\alpha-2\alpha(1-\gamma_j)]$, the best response of firm i is $b_i^{I}(\gamma_i) = \frac{1}{6}[2-\alpha-2\alpha(1-\gamma_i)]$. ■

Proof of proposition 5: In a second price sealed bid auction the ex-post revenue is same as the second highest bid. So, given the equilibrium bids $b_i^{II}(\gamma_i) = \frac{1}{3}[1-2\alpha(1-\gamma_i)]$, (i = 1, 2) the expected revenue in the second price auction is given by $ER^{II} = \frac{1}{3}[1-2\alpha\{1-E(\gamma_L)\}]$, where $\gamma_L = \min\{\gamma_1, \gamma_2\}$ is the *second highest order statistic* in our case[14]. Since γ_L is the second highest order statistic drawn from the distribution of γ_i, which follows uniform distribution in the interval [1/2, 1], the distribution of γ_L is given by $F(\gamma_L) = 1 - 4(1-\gamma_L)^2$ and hence the density function is $f(\gamma_L) = 8(1-\gamma_L)$. Therefore,

$$E(\gamma_L) = \int_{½}^{1} 8(1-\gamma_L)\gamma_L d\gamma_L = \frac{2}{3}.$$

Substituting the value of $E(\gamma_L)$ in the in the expected revenue function ER^{II}, we get,

$$ER^{II} = \frac{1}{3}\left(1 - \frac{2\alpha}{3}\right).$$

In a first price sealed bid auction the ex-post revenue is same as the highest bid and given the equilibrium bids $b_i^I(\gamma_i) = \frac{1}{6}[2 - \alpha - 2\alpha(1-\gamma_i)]$, the expected revenue is $ER^I = \frac{1}{6}[2 - \alpha - 2\alpha\{1 - E(\gamma_H)\}]$, where $\gamma_H = \max\{\gamma_1, \gamma_2\}$ is the *highest order statistic*. The distribution of γ_H is given by $G(\gamma_H) = 4(\gamma_H - ½)^2$ and hence the density function is $g(\gamma_H) = 4(2\gamma_H - 1)$. Therefore,

$$E(\gamma_H) = \int_{½}^{1} 4(2\gamma_H - 1)\gamma_H d\gamma_H = \frac{5}{6}.$$

Substituting the value of $E(\gamma_H)$ in the in the expected revenue function ER^I, we get,

$$ER^I = \frac{1}{3}\left(1 - \frac{2\alpha}{3}\right).$$ ■

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[14] Let $X_1, X_2, \ldots, X_n$ be n independent draws from a distribution F with associated density f. Let $Y_1, Y_2, \ldots, Y_n$ be a rearrangement of those such that $Y_1 \geq Y_2 \geq \ldots \geq Y_n$. The random variable Y_k, k = 1, 2, … , n is referred to as the k th order statistic. See Krishna (2002) for further details.

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