Solving the Moral hazard with Rating Agencies

Abstract: Credit Rating agencies suffer from a possible moral hazard problem. This is caused due to the fact that the evaluation standard set by rating agencies are unobservable to outsiders. In this paper, we address the issue of possible moral hazard that rating agencies might have. We discuss the feasibility of possible incentive contracts that can ameliorate this problem. We find, that incentive payments to the rating agency based on expected returns on debt will do away with the moral hazard problem.

JEL Classification No. **G140, G200, G290**

Key Words: Credit Rating, Information Production, Moral Hazard.

1 Introduction

Rating agencies are unique because, the users of their services do not pay directly for the information. Credit rating agencies evaluate and rate debt instruments. They publish ratings on the riskiness of such instruments. The firm, whose instruments are rated, pay for these services, while the ratings are available as public information, and hence, the users of these ratings, the investors, do not directly pay. Another, interesting fact associated with the ratings industry is that it is a regulated oligopolistic industry all over. In India, bulk of the ratings are done by the two leading rating agencies in India, CRISIL (Credit Rating Information Services of India Limited) and ICRA (Investment Information and Credit Rating Agency of India Limited). Often the arguments in favor of having such a few rating agencies are that, they promote 'unhealthy competition'. These two facts lead to an obvious problem of *moral hazard*. In this paper, we discuss incentive contracts that will tackle such moral hazard issues. We model the credit rating agencies as information producing financial intermediaries.

We consider a simple model where the economy consists of four risk neutral agents - an investor, a firm, a credit rating agency and the regulator (the government). The firm has a project with uncertain returns requiring fixed initiation costs. The firms have private information regarding the default probability of their projects. The firm does not have capital to initiate the project and hence, has to approach the investor. This investment is raised by offering debt. The investor is rational and has Bayesian beliefs. The rating agency charges a fee to the firms that come to it to be rated. It then evaluates the firms using its screening technology. Information production by a rating agency depends upon the screening function it has and the evaluation standard it sets. This function is such that the accuracy with which the rating agency can infer a firm's type, increases with stringent evaluation standard. The fee is charged ex ante and is same across all the types. The screening function used by the rating agency distinguishes across types with less than perfect precision. The precision level can be improved by the rating agency if it sets higher evaluation standards. However, as setting stricter evaluation standard is costlier and this cost is strictly unobservable to third parties, the rating agencies are characterized by moral hazard problems. A crucial distinction is made regarding the unobservable evaluation standard. The rating agency may, after its evaluation of a firm based on the evaluation standard it has set, conclude it to be not investment worthy. however, it may be in its own interest to misreport its findings. Thus, it may certify the firm as investment worthy while it actually found out that the firm was not. We rule out such possibilities in this paper. The reason being, the conclusions arrived at about a firm, stays on the records of the rating agency, and hence, can be used in the court should a future dispute arise. This misreporting tantamount to fraudulent practises which is ruled out by assumption. However, the rating agency can actually set lenient evaluation structure than what it claims. This is because, the evaluation standard set by it is unverifiable and hence, cannot be used in the court of law. In this paper, we tackle this moral hazard problem-namely, the scenario where the rating agency claims it has set a particular evaluation standard, but actually sets something that is quite different.

There has been a substantial empirical study, on the effectiveness of rating agencies. Partnoy (2001), provides a brief survey of empirical work till date. Empirical studies by Ederington, Goh and Nelson (1996) compares the information effectiveness of rating agencies and stock market analysts on market movement and efficiency. White (2001) also study the effectiveness of bond ratings and proposes welfare implications. Partnoy (2001), tests the hypothesis that ratings are effective, although the market may actually anticipate it in advance and hence, ratings are published with a lag. Recent attempts in theoretical modelling of rating agencies, start with Nayar (1993). In his paper, Nayar establishes the case for voluntary ratings as against compulsory ratings for the Malaysian firms. Kuhner (2001) develops a model that identifies the likely scenarios where the investor may completely disregard or base their decisions based on the ratings. He does identify a separating Bayesian Nash equilibrium where some meaningful information by the rating agency is disseminated and used by the investors. In another interesting recent paper, Boot and Milbourn (2001) show that rating agencies usually act as a mechanism that coordinates to 'focal points'.[1]

The remainder of the paper is organized as follows. In section 2, we present the benchmark model. We first solve the model with the case that the evaluation standard is observable. this is done in section 3. In section 4, we consider the moral hazard problem. Finally 5 concludes. All the proofs of the results being relegated to Appendix in 6.

2 Benchmark Model of Information Production

There are four sets of risk neutral agents - an investor, a firm, a credit rating agency (CRA) and the government (who has the regulatory powers).

The investor, I, is endowed with capital. She can either invest a part of this in the risk free asset or can invest in the firm. For simplicity, assume that the risk free rate is r.

The firm, has a project that requires one unit of capital as input today. The project produces uncertain return streams of S_T when it matures, T periods from now, $S_T \in [0, \infty)$. The uncertainty in the project cash flows are summarized by the probability distribution, $G(S_T)$. The firm has no funds available to finance the project. Therefore, the firm has to raise the required amount of one unit from the investor if the project has to be started. The firm resorts to debt financing. The debt contract involves a face value of D per unit of capital borrowed payable at T. We further assume that this debt is like a *zero coupon bond*, that rules out interim payments.[2] Given limited liability for the firm, D is paid in full iff $S_T \geq D$. For any $S_T < D$, debt being senior to equity, entire S_T is appropriated by the investor. The government taxes the investor on her debt earnings and the firm, on his

[1] Apart from financial intermediation, the role of agencies or 'experts' as information producers have also gained prominence. Biglaiser (1993), Albano and Lizzeri (1997) model the role of experts who evaluate product qualities. These models have similar framework to ours but they study very different problems.

[2] Although not crucial to the argument, a zero coupon bond simplifies the analytical model a great deal.

profits (equity) at a uniform rate of t. Therefore, the returns to the firm is $(1-t)Max\{S_T - D, 0\}$ and to the investor is $(1-t)Min\{S_T, D\}$.

As the debt matures at T and there are no interim payments, the value of debt , V_D is,

$$V_D = e^{-rT}(1-t)\left\{\int_0^D S_T dG(S_T) + \int_D^\infty D dG(S_T)\right\}.$$

A.1: S_T follows a generalized *Weiner process* with a drift rate of μ and standard deviation σ.

$$dz = z\left\{\mu dt + \sigma\epsilon\sqrt{dt}\right\}. \quad (1)$$

In the above equation ϵ is a *White noise* such that $\epsilon \sim Normal[0,1]$.

We assume S_T follows *Log normal* distribution with mean $(\mu + \frac{\sigma^2}{2}T)$ and an annual volatility (in percentage terms) of $\sigma\sqrt{T}$.[3] The volatility, $\sigma\sqrt{T}$ is type specific to the firm. Note that, as T is common knowledge, effectively, σ is type specific. We will assume that, $\sigma \in [0, \bar{\sigma}]$.

Let S_0 be the current value (all equity) of the firm. Then,

$$ln(S_T) \sim \phi\left[ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]. \quad (2)$$

Therefore, the value of the debt, $V_D(\sigma)$, with a standard deviation of σ is,

$$\begin{aligned} V_D(\sigma) &= \frac{1-t}{\sqrt{2\pi}\sigma\sqrt{T}}\int_0^D e^{-\frac{1}{2\sigma^2 T}\left[ln(S_T)-\left(r-\frac{\sigma^2}{2}\right)T\right]^2} dS_T \\ &+ \frac{D(1-t)}{\sqrt{2\pi}\sigma\sqrt{T}}\int_D^\infty \frac{1}{S_T} e^{-\frac{1}{2\sigma^2 T}\left[ln(S_T)-\left(r-\frac{\sigma^2}{2}\right)T\right]^2} dS_T. \end{aligned}$$

The value of equity is,

$$V_E(\sigma) = (1-t)\{S_0\Phi(d_1) - De^{-rT}\Phi(d_2)\} \quad (3)$$

and the value of the debt is,

$$V_D(\sigma) = (1-t)\{S_0[1-\Phi(d_1)] + De^{-rT}\Phi(d_2)\}, \quad (4)$$

[3]This is a standard assumption for modelling stock price behaviour.

where

$$d_1 \equiv \frac{ln\left(\frac{S_0}{D}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}.$$

In the above expression, $\Phi(x)$ denotes the cumulative Normal probability distribution corresponding to x. Subsequently, we will denote $\phi(x)$ as the probability density function corresponding to x.

Lemma 1 *$V_E(\sigma)$ increases with σ while $V_D(\sigma)$ decreases with σ.*

From the equations (3) and (4) it is immediate that,

$$\begin{aligned}
&V_E(0) = S_0 - De^{-r.T} \\
&V_D(0) = De^{-r.T} \\
&Lim_{\sigma\to\infty}V_E(\sigma) \to S_0 \\
&Lim_{\sigma\to\infty}V_D(\sigma) \to 0.
\end{aligned}$$

Note that, lemma 1 guarantees that $\exists\tilde{\sigma}$ such that, $V_D(\tilde{\sigma}) = 1$. To the investor, all those types are 'good' which has $\sigma \leq \tilde{\sigma}$.

Throughout the remainder of the paper, we assume that in the absence of any additional information that distinguishes the types, the investor will not invest in the firm. As $V_D(\sigma)$ is decreasing in σ, this implies,

A.2 :

$$\int_0^{\bar{\sigma}} V_D(\sigma)dF(\sigma) < 1 \Leftrightarrow \sigma^e \equiv E(\sigma) > \tilde{\sigma}.$$

However, as σ is type specific, the investor only knows the distribution, $F(\sigma)$ from which σ is drawn. She therefore, has to rely on a third party, the CRA, to provide additional information about the firms' σ.

A.3: $F(\sigma)$ is uniformly distributed over $[0, \bar{\sigma}]$ with $F(0) = 0, F(\bar{\sigma}) = 1$ and a density function, $f(\sigma)$.

The CRA(C) conveys additional information about the firm's actual type to the investors. The CRA charges an evaluation fee of X for its rating services. This fee is paid to it by the government, G. This is as if, when the CRA applies for the license, G pays it a net fee of X.[4] The government subsequently taxes the investor and the firm adequately to recover X.

[4]We use net to indicate the difference between what the CRA is paid by G and what it pays as the possible licensing fees.

The CRA evaluates the firm with the help of the technology it has and the *evaluation standard* it sets. The accuracy with which it can distinguish the firm's success rate p, depends upon the evaluation standard, e, it sets. For simplicity we assume that $e \in [0, 1]$.

Finally, the CRA announces its findings about the firm to the investor. The CRA announces the firm to be 'good' if it concludes that the firm's $\sigma \leq \tilde{\sigma}$ and 'bad' if $\sigma > \tilde{\sigma}$. The investor has Bayesian beliefs, and updates her priors regarding the firm's type, conditional on the announcements made about the firm. The decision whether or not to invest in the firm, depends upon these announcements. Given any announcement, a by the CRA, the investor invests if it is profitable for her to do so. An announcement 'good' will be denoted by 'g' and 'bad' will be denoted by 'b'. [5]

The government, G plays the role of a regulator. As a regulator, the government has to ensure that the *moral hazard* problem with the CRA is minimum. We assume throughout that, X is observable and exogenous to the model.

Technology: The technology available with the CRA generates output in the form of reports, $r(\sigma)$ that classify a firm having σ either greater than or less than $\tilde{\sigma}$. The accuracy with which a type is inferred correctly depends upon e. A stricter evaluation standard (higher e) allows it to infer the actual type of a firm more accurately. Corresponding to e, $r(\sigma)$ is such that, $0 \leq l(\sigma; e) \leq r(\sigma) \leq u(\sigma; e) \leq \bar{\sigma}$. The technology and e determines $l(\sigma; e)$ and $u(\sigma; e)$. If $r(\sigma) \leq \tilde{\sigma}$, then the CRA concludes it to be good. Similarly, if $r(\sigma) > \tilde{\sigma}$ it concludes the firm to be bad. At the extreme, $e = 1$ corresponds to the "perfect" inference case. Note with $e = 1$, the perfect inference case, $l(\sigma; 1) = u(\sigma; 1) = r(\sigma) = \sigma$. With $e = 0$, $l(\sigma, 0) = 0$ and $u(\sigma, 0) = \bar{\sigma}$. Therefore, if $e = 0$ then any type can generate a report between $[0, 1]$ with equal probability. Therefore, with $e = 1$ the technology is perfect. That is, the CRA knows the exact σ. On the other extreme, with $e = 0$, the technology does not convey any additional information regarding the firm's type other than the fact that σ is drawn from $F(\sigma)$ and lies between 0 and $\bar{\sigma}$. Different values of e can be attributed to the differences in actual parameters CRAs may wish to investigate. In our model the evaluation standard is *uni-dimensional.* We make e uni dimensional as it considerably simplifies the algebra. Further, one can interpret e having weighted components of all the parameters different agencies wish to investigate. We assume away any

[5] In reality the announcements are multi-dimensional. However, to avoid unnecessary algebraic complications, we restrict ourselves to the case where the CRA just announces 'good' or 'bad'. In practice, the CRA classifies firms into categories starting from a 'high risk - speculative grade' to 'low risk - investment grades.'

strategic announcements by the CRA. That is the CRA always announces according to its announcement rule. In other words, given a choice of unverifiable e, if $r(\sigma, e) > \tilde{\sigma}$ then the CRA cannot announce the firm as good. Outsiders can easily verify $r(.)$ (although not e) and hence the CRA will not 'cheat'. Verifiability of $r(.)$ is not unusual. Note that, r can be interpreted as the result of the CRA's inference of σ, which may or may not be close to the actual σ, a fact that is determined by e. This effectively means that r is the report submitted by the auditors to the CRA about a firm. The CRA can be legally held responsible if it 'hides' the auditors' report and publishes something else.

Let $\alpha(\sigma, e) \in [0, 1]$ denote the probability that any particular type will lead to an announcement 'good'. The probability with which any type is announced good is therefore simply the probability that $r \leq \tilde{\sigma}$. For, $F(\sigma)$, $\alpha(\sigma, \tilde{\sigma}, e)$ is given by

$$\begin{aligned}
\alpha(\sigma, \tilde{\sigma}, 1) &= \begin{cases} 1 & \text{if } \sigma \leq \tilde{\sigma}; \\ 0, & \text{if } \sigma > \tilde{\sigma} \end{cases} \\
\text{and} \quad & \forall \quad e \in [0, 1), \\
\alpha(\sigma, \tilde{\sigma}, e) &= \begin{cases} 1 & \text{if } F(u(\sigma, e)) < F(\tilde{\sigma}); \\ \frac{F(\tilde{\sigma}) - F(l(\sigma, e))}{F(u(\sigma, e)) - F(l(\sigma, e))} & \text{if } F(u(\sigma, e)) \geq F(\tilde{\sigma}) \geq F(l(\sigma, e)) \\ 0 & \text{if } F(\tilde{\sigma}) \leq F(l(\sigma, e)) \end{cases}
\end{aligned} \tag{5}$$

For example, if σ follows *Uniform distribution*, then $l(\sigma, e) = \sigma e$ and $u(\sigma, e) = \sigma e + \bar{\sigma}(1 - e)$.

$$\begin{aligned}
\alpha(\sigma, \tilde{\sigma}, 1) &= \begin{cases} 1 & \text{if } \sigma \leq \tilde{\sigma}; \\ 0, & \text{if } \sigma > \tilde{\sigma} \end{cases} \\
\text{and} \quad & \forall \quad e \in [0, 1), \\
\alpha(p, \tilde{\sigma}, e) &= \begin{cases} 1 & \text{if } \bar{\sigma}(1 - e) + e\sigma < \tilde{\sigma}; \\ \frac{\tilde{\sigma} - e.\sigma}{\bar{\sigma}(1-e)} & \text{if } \bar{\sigma}(1 - e) + e\sigma \geq \tilde{\sigma} \geq e\sigma \\ 0 & \text{if } \tilde{\sigma} \leq e\sigma \end{cases} .
\end{aligned}$$

3 Observable Evaluation Standards.

We first present the equilibrium results as if e is observable. Thereafter, we allow for unobservable e in section 4.

Sequencing: The agents in the model move in the following sequence.
• In the first stage, the regulator issues an operating license to the CRA and makes a net payment of X to it. The terms of the license (contract) relates an incentive payment structure to the evaluation standard the CRA

claims to have set. The CRA claims that once given the license, it will set an evaluation standard of m with $0 \leq m \leq 1$.

• In the second stage, the CRA sets its evaluation standard e. Possibility of $e \neq m$ is the source of moral hazard.

• In the third stage, the firm decides whether or not to get rated by the CRA. If it decides not to get rated, it will approach the investor for funding.

• In the fourth stage, the CRA evaluates the firm and then publishes this rating. Access to this ratings are publicly available to all the parties.

• The investor decides to invest in the firm based on the announcement made by the CRA. Depending upon the investor's decision, the project is either taken up or not taken up by the firm.

• At T payments are made out of the project earnings. The investor and the firm also pays the requisite taxes to the government, while the government in turn, pays the incentive component to the CRA as agreed upon by both of them during signing of the contract at the first stage.

The following variables are endogenously determined in the model. The government determines the fixed component part of the fees, X and the incentive component part. The CRA decides upon the the two evaluation standards m and e. While the firm decides whether or not to get rated, the investor decides whether or not she should invest given the announcements. We will assume that the tax rate t is exogenous as we are not interested in determining whether the government can achieve *budget balancedness*.[6]

Investor's Decision: Denote $\Lambda = \{\gamma(e), \beta(e)\}$ such that, $\gamma(e)$ and $\beta(e)$ are the probability with which the investor invests if the firm is announced good and bad, respectively. Therefore, $\gamma(e), \beta(e) \in [0, 1]$. The investor calculates, $E(V_D|a = g)$ and $E(V_D|a = b)$. If the firm is announced 'good', she will lend to him iff $E(V_D|a = g) \geq 1$. Similarly, when the firm is announced 'bad', she will lend to him iff $E(V_D|a = b) \geq 1$. Let φ be the probability with which the investor invests in the firm if he approaches her directly.

Firm's Decision : If the firm goes to the CRA, the expected profit he earns is,

$$\Pi_\sigma = V_E(\sigma)\{\alpha(\sigma, e)\gamma(e) + [1 - \alpha(\sigma, e)]\beta(e)\}. \tag{6}$$

The RHS of the above equation is made up as follows. The firm makes a positive expected profits of $V_E(\sigma)$ if and only if the investor invests in him. This can arise in either of the two ways. One, when it is announced 'good'

[6]One can construct appropriate values of t that will ensure that the government does not incur budgetary deficit and all the results hold true.

with a probability of $\alpha(\sigma, e)$. The investor then invests with a probability $\gamma(e)$. Two, when the firm is announced bad with probability $1-\alpha(\sigma, e)$ and the investor invests with a probability of $\beta(e)$ even when she knows that the announcement is bad.

The above equation indicates that it is feasible for all the types to go to the CRA as it costs him nothing. However, whether all the types go to the CRA or not would depend upon the fact that, whether for any particular type, approaching the investor directly for funds is more profitable. Denote, Π_σ^0 as the expected profit to the firm if he approaches the investor directly for funds. Let φ is the probability with which the investor invests in a firm that comes to her directly. Therefore,

$$\Pi_\sigma^0 = V_E(\sigma)\varphi.$$

However, as the result below establishes, the firm will never approach the investor directly for funding.

Proposition 1 *All types find it worthwhile to go to the CRA.*

The expected probability of success corresponding to any announcement is given by

$$E(V_D|\text{a}=\text{g}) = \frac{\int_0^{\bar{\sigma}} V_D(\sigma).\alpha(\sigma, e)dF(\sigma)}{\int_0^{\bar{\sigma}} \alpha(\sigma, e)dF(\sigma)}.$$

$$E(V_D|\text{a}=\text{b}) = \frac{\int_0^{\bar{\sigma}} V_D(\sigma)[1-\alpha(\sigma, e)]dF(\sigma)}{\int_0^{\bar{\sigma}} [1-\alpha(\sigma, e)]dF(\sigma)}.$$

Therefore,

$$E(V_D|\text{a=g}) \geq 1 \;\Rightarrow\; \int_0^{\bar{\sigma}} (V_D(\sigma)-1)\alpha(\sigma, e)dF(\sigma) \geq 0. \tag{7}$$

$$E(V_D|\text{a=b}) \geq 1 \;\Rightarrow\; \int_0^{\bar{\sigma}} (V_D(\sigma)-1)[1-\alpha(\sigma, e)]dF(\sigma) \geq 0. \tag{8}$$

CRAs' Decisions: The CRA sets e and incurs a cost of $c(e)$. Therefore, its expected profits are,

$$X - c(e). \tag{9}$$

The main result in this section is organized as follows. In the proposition below, we show that there exists a unique *Pure Strategy Bayesian Nash equilibrium.*

Proposition 2 *Under A.1-A.3, $\exists$ a unique Bayesian Nash equilibrium $\{e^*, \Lambda^*\}$ such that,*

$$(a) \quad E(V_D|a=g) = 1 > E(V_D|a=b) \Rightarrow \gamma^* = 1; \beta^* = 0$$
$$(b) \quad \int_0^{\bar{\sigma}} [V_D(\sigma) - 1]\alpha(\sigma, e^*)dF(\sigma) = 0 \quad \textit{and} \quad 0 < e^* < 1.$$

Part (a) of above indicates that, the equilibrium evaluation standard is less than perfect but not zero. We have $e^* > 0$ because, otherwise, with $e^* = 0$, the CRA is unable to distinguish across types. Therefore, it cannot convey any additional information through its' ratings. However, $e^* < 1$ because, setting higher e is costly for the CRA. Therefore, it will set the minimum e such that the investor breaks even. The crucial requirement is that the technology is able to distinguish across types and thereby different types are announced 'good' with different probabilities. Thus we observe that, the Bayesian Nash equilibrium is unique and involves only pure strategies.

4 Unobservable Evaluation Standards.

In this section we will show the existence and the characteristics of an incentive contract such that, any CRA that claims it has set an equilibrium evaluation of e^*, indeed sets $e = e^*$. Once such a contract is in place, the results in the preceding section section holds true. This is because, this contract becomes common knowledge and the investor knows with certainty that the CRA has indeed set $e = e^*$. Therefore, we will only be interested in characterizing the incentive contract.

Let $m \in [0, 1]$ be the CRA's claim that it will set an evaluation standard of m. In this paper, we are interested in designing the truth revealing incentive contract. In other words, we show existence of contracts such that, if the CRA claims it has set $m = e^*$, it actually sets $e = e^*$. Such a contract will specify a transfer payment by the regulator to the CRA based on the announcements by the latter. Two factors are important in designing such a contract. The first factor is the time when the payments are to be made according to the contract. That is, should the payments be made during the time when the CRA seeks the license or after it has rated a firm. The second factor is the component on which the incentive payments have to be linked- the value of debt, the value of equity or both.

The first result shows the impossibility of designing a contract where the incentive payments have to be paid before the project is undertaken.

Proposition 3 *The truthful revelation contract cannot have the incentive component linked only to the ex ante announcements by the CRA.*

The above result suggests that, for an incentive contract that is truth revealing, the incentive payments are have to be made conditional on the project realizations. We explore two such possible contracts below.

let Δ and Ω are two contracts such that Δ is linked to the value of debt and Ω is linked to the value of equity. Note that, at the time of designing Δ and Ω, the values of debt and equity are unknown. Therefore, the payments are linked to the expected values of debt and equity.

Denote $\Pi_c^k(m,e), k = \Delta, \Omega, \forall m, e \in [0,1]$ as the expected profit to the CRA under the contracts Δ and Ω when it claims m and sets e. Once the CRA sets e, the only verifiable information to the outsiders is $r(\sigma)$ and the announcements made on the basis of $r(\sigma)$. Recall, $r(\sigma)$ is the report generated by the firm to the CRA. However, as e is not verifiable, the regulator has to design an incentive contract such that the true choice of e is revealed. This contract can be based on the verifiable component m. Let the total payment made by G to C under the terms of the incentive contract is $X + \phi(m)$. Here X is the same as before, while $\phi(m)$ captures the payments made contingent on the evaluation standard, the CRA claims to set.

We consider two such contracts. In the first contract, the regulator taxes the firm *ex post* at a unit rate t and pays a certain proportion $\theta_\Delta(m)$ of its collection to the CRA. In the second contract, the regulator taxes the investor *ex post* at a unit rate t and pays a certain portion, $\theta_\Omega(m)$ to the CRA. In other words, $\phi(m) = \{t\theta_\Delta V_D, t\theta_\Omega V_E\}$. It is evident that, $\theta_k, k = \Delta, \Omega$ can be made conditional only on m.[7]

Note that,

$$
\begin{aligned}
\Pi_c^\Delta(m,e) &= X - c(e) \\
&+ \theta_\Delta(m)t\int_0^{\bar{\sigma}} V_D(\sigma)\left\{[\gamma(m) - \beta(m)]\alpha(\sigma,e) - \beta(m)\right\} dF(\sigma) \\
\Pi_c^\Omega(m,e) &= X - c(e)
\end{aligned}
$$

[7]One can do away with the regulator altogether in designing the contracts. In other words, the investor or the firm may directly promise the contracts Δ and Ω respectively, such that θ_Δ and θ_Ω is the transfer payment by the investor and the firm respectively, to the CRA. However, we involve the government/ regulator to design the contract as that would be more viable in the real world.

$$+ \quad \theta_\Delta(m)t \int_0^{\bar{\sigma}} V_E(\sigma) \left\{[\gamma(m) - \beta(m)]\alpha(\sigma, e) - \beta(m)\right\} dF(\sigma).$$

In the above expressions, the sources of revenue to the CRA are X the revenue form the firm and the incentive component term(where for $k = \Delta, j = D$ and for $k = \Omega, j = E$),

$$\theta_k(m)t \int_0^1 V_j(\sigma) \left\{[\gamma(m) - \beta(m)]\alpha(\sigma, e) - \beta(m)\right\} dF(\sigma).$$

Note that, γ and β, the probabilities, with which the investor invests, is contingent only upon the contractible component m. For any $e_1 > e_2$, truthful revelation of the *hidden action* e implies, with contract Δ

$$\begin{aligned} & \Pi_c^\Delta(e_1, e_1) > \Pi_c^\Delta(e_1, e_2) \quad \text{and} \quad \Pi_c^\Delta(e_2, e_2) > \Pi_c^\Delta(e_2, e_1) \Rightarrow \\ (A) \quad & t\theta_\Delta(e_1) \int_0^{\bar{\sigma}} V_D(\sigma)[\gamma(e_1) - \beta(e_1)][\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma) \\ & \qquad > c(e_1) - c(e_2) \geq \\ & t\theta_\Delta(e_2) \int_0^{\bar{\sigma}} V_D(\sigma)[\gamma(e_2) - \beta(e_2)][\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma) \end{aligned}$$

and with contract Ω,

$$\begin{aligned} & \Pi_c^\Omega(e_1, e_1) > \Pi_c^\Omega(e_1, e_2) \quad \text{and} \quad \Pi_c^\Omega(e_2, e_2) > \Pi_c^\Omega(e_2, e_1) \Rightarrow \\ (B) \quad & t\theta_\Omega(e_1) \int_0^{\bar{\sigma}} V_E(\sigma)[\gamma(e_1) - \beta(e_1)][\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma) \\ & \qquad > c(e_1) - c(e_2) \geq \\ & t\theta_\Omega(e_2) \int_0^{\bar{\sigma}} V_E(\sigma)[\gamma(e_2) - \beta(e_2)][\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma). \end{aligned}$$

Solution to the above set of inequalities will give the condition for the *separating equilibrium.*

Proposition 4 *Contract Ω cannot be truth revealing.*

Therefore, if there indeed exists a possible truthful revelation equilibrium, it is only possible with the incentive contract Δ. The next result shows the existence of a truthful revelation equilibrium.

Proposition 5 *Under A.1-A.3, $\exists\tilde{\sigma}^*$ such that for $\tilde{\sigma} \leq \tilde{\sigma}^*$, no truth revealing equilibrium exists. However, for $\tilde{\sigma} > \tilde{\sigma}^*$, with a low marginal cost of setting evaluation standards, there exists a truthful revelation where $\theta^*_\Delta(m)$ is increasing in the announcement m.*

Thus, for sufficiently large $\tilde{\sigma}$ (that is, $\tilde{\sigma} \geq \tilde{\sigma}^*$, and small marginal cost of setting higher evaluation standard, that is, $c(e_1) - c(e_2) = \epsilon$ where ϵ is small but positive, we can define a function $\theta_\lambda(m)$ such that it is truth revealing.

Denote $\theta \equiv \theta(e_i), V_D(\sigma) \equiv V_D; c_i \equiv c(e_i)$. The continuous equivalent of the above problem can be stated as follows. Rewrite (A) as

$$\theta_1 \int_0^{\bar{\sigma}} V_D[\alpha_1 - \alpha_2] dF(\sigma) > \frac{c_1 - c_2}{t} \geq \theta_2 \int_0^{\bar{\sigma}} V_D[\alpha_1 - \alpha_2)] dF(\sigma).$$

Letting, $e_1 = e_2 + h$, and taking the limit $h \to 0$ on both sides, we obtain,

$$\theta^*(m) = c'(m) \left[t \int_0^{\bar{\sigma}} V_D \frac{\partial \alpha(\sigma, e)}{\partial e} |_{e=m} dF(\sigma) \right]^{-1}.$$

Also X^* solves, $X^* = c(e^*) - \theta^*(e^*)$.

The results in this section presents a few interesting points. It shows that, the government can design a contract that will compel the CRA to choose the evaluation standard which the CRA claims to have set in the first stage. This contract is simple. It contains an incentive component, that positively links the payments made to the CRA with two items. One, the evaluation standard it claims to have set, and two, the ex post value of the debt.
(a) The incentive payment to the CRA has to be linked with the project realizations. Therefore, the payments have to be made *ex post*. The reason for this is easy to see. In order to induce truth telling, the incentive payments should be linked to its performance. Although the performance, i.e., the evaluation efficiency, is not directly contractible, the returns to the various agents (the investor and the firm) are. Thus, the incentive payments have to be linked to these returns.
(b) The incentive contract has to be linked with the value of the debt and not the equity. However, if the average riskiness of the projects are very high compared to what the investor perceives as a good project, moral hazard cannot be ruled out even with an incentive contract linked to debt.

Why does equity linked incentive contract does not work? This is primarily because, if the payments are made contingent on the equity earnings, the CRA will have an incentive to promote riskier projects as the equity returns are higher as the risk is higher. However, if the payments are linked with debt value then the CRA knows that if it tries to promote firms with higher σ, the CRA will earn less in expected terms from the incentive contract. This is because, of the fact that the expected returns linked with the value of debt, increases as σ decreases. Thus in order discourage the CRA from promoting risky projects, the incentive component must be linked to the value of debt. However, if the gap between σ^e and $\tilde{\sigma}$ is significant, i.e, if the measure of good firms are already very low, the CRA knows, going by a stricter evaluation standard it can announce fewer good types to be investment worthy. It is then that the CRA will set an evaluation standard lower than what it claims it will set.

5 Conclusion

The rating agencies by the way of its' operations and the structure of the industry, are subject to moral hazard problems. These problems arise because, the rating agencies can claim to set higher evaluation standards, that minimizes the type I and type II errors, but actually set much lower evaluation standards. This happens because setting stricter evaluation standards are costly. Their incentive to do so arises mainly from two facts. One, the users of their services do not directly pay, and two, the industry is characterized by oligopoly with very little competition. The rating fees are paid by the firm, and naturally, the rating agencies are inclined more towards the firms' interests, as damaging ratings will in turn lead to a lower probability that a firm will get rated by the agency.

In this paper, we present a simple model where the investors, with Bayesian beliefs, have sufficient funds to fund a project possessed by the firm. The project has stochastic returns with the riskiness factor being private knowledge to the firm. The investor relies on the rating agencies ability to disseminate the information about the riskiness of the project.

We find that, a regulator can eliminate this moral hazard problem by an appropriate contract designing. The contract has specifies a payment to the rating agency from the tax earnings of the firm/ investor based on the evaluation standard the rating agency claims it has set. We find that, in equilibrium,

• The incentive payment to the CRA has to be linked with the project realizations. Therefore, the payments have to be made *ex post*, i.e., out of the project earnings.

• The incentive contract has to be linked with the value of the debt and not the equity.

• In case the incentive structure is linked to both, the component related to debt must not only exceed that with equity, but exceed the component linked to debt when the incentive structure is linked to debt alone.

6 Appendix

• **Proof of Lemma 1.**

Proof: Note that,

$$\vartheta = \frac{\partial V_E(\sigma)}{\partial \sigma} = (1-t)S_0\phi(d_1)\sqrt{T} > 0.$$

This is the standard 'Vega' expression for an European non dividend paying call option. V_D decreases in σ follows from the fact that $S_0 = V_D + V_E$. Therefore,

$$\frac{\partial V_D(\sigma)}{\partial \sigma} = -(1-t)S_0\phi(d_1)\sqrt{T} < 0.$$

■

• **Proof of Proposition 1.**

Proof: Let σ_m be such that, $\Pi_{\sigma_m} = \Pi^0_{\sigma_m}$. That is, σ_m is the type that is indifferent between going to the CRA and going to the investor directly. We will show that no types will come to the investor directly.

$$\Pi_{\sigma_m} = \Pi^0_{\sigma_m} \Rightarrow \alpha(\sigma_m, e)[\gamma(e) - \beta(e)] + \beta(e) = \varphi.$$

As $\alpha(\sigma, e)$ is non increasing in σ, it is obvious that $\forall\sigma > \sigma_m$, $\Pi^0_\sigma > \Pi_\sigma$. Therefore, the investor correctly infers that all types with $\sigma > \tilde{\sigma}$ will come to it directly for investment. As $V_D(\sigma)$ is decreasing in σ, this implies that the investor will never invest in any type that comes to her directly. Therefore any firm that goes directly to the investor earns zero expected profits. By going to the CRA the firm can expect to earn non negative expected profits. Therefore, all types will go to the CRA. ■

The following lemma will be useful in proving some of the results.

Lemma 2 *The following conditions are true,*
$\forall e \in [0,1], \quad E(V_D(\sigma)|a = g) \geq E(V_D(\sigma)|a = b)$
(ii) $\forall\tilde{\sigma}, \quad \exists e \in [0,1]$ *such that* $E(\sigma|a = g) \leq \tilde{\sigma}$.

Proof:

(i) This is obvious given that, $\alpha(\sigma, e)$ decreases with σ.

(ii) Note that, at $e = 1$, $\alpha(\sigma, 1) = 1\forall\sigma \leq \tilde{\sigma}$ and $\alpha(\sigma, 1) = 0$ for all $\sigma > \tilde{\sigma}$. This immediately implies, at $e = 1$, $E(\sigma|a = g) \leq \tilde{\sigma}$. ■

Define
$$H(\tilde{\sigma}, e) \equiv \int_0^{\bar{\sigma}} (V_D(\sigma) - 1)\alpha(\sigma, e)dF(\sigma).$$
Note that $H(.)$ is obtained from solving $E(V_D(\sigma)|a = g) \geq 1$. This is given in equation (7). The investor's decision to evaluate the announcement made by the CRA depends upon the sign of $H(\tilde{\sigma}, e)$.

Lemma 3 $H_e \geq 0$.

Proof:
$$\begin{aligned} H(\tilde{\sigma}, e) &= \int_0^{\tilde{\sigma}} (V_D(\sigma) - 1).\alpha(p, e)dF(\sigma) + \int_{\tilde{\sigma}}^{\bar{\sigma}} (V_D(\sigma) - 1).\alpha(p, e)dF(\sigma) \\ H_e &= \int_0^{\tilde{\sigma}} (V_D(\sigma) - 1).\alpha_e(p, e)dF(\sigma) + \int_{\tilde{\sigma}}^{\bar{\sigma}} (V_D(\sigma) - 1).\alpha_e(p, e)dF(\sigma). \end{aligned}$$
Given the properties of $\alpha(\sigma, e)$, it is immediate that $H_e \geq 0$. ■

• **Proof of Proposition 2.**

Proof: (a) We first show that $\gamma(e) \geq \beta(e)$ for all e. From the properties of $\alpha(\sigma, e)$, it is easy to see that $E(V_D|a = g) \geq E(V_D|a = b)$. This follows from the fact that $\alpha(\sigma, e)$ is non increasing in σ.

We now show that in equilibrium,
$$E(V_D|a = g) = 1 > E(V_D|a = b).$$

Suppose, $E(V_D|a = g) \neq 1$. There are two possibilities. Let $E(V_D|a = g) < 1$. That is, the investor does not invest when the announcement is good. Then from the fact that $E(V_D|a = g) \geq E(V_D|a = b)$, she does not invest when the announcement is bad. This is equivalent to state that the investor does not take into consideration the announcements by the CRA. The CRA cannot operate profitably then. The CRA can always set $e = 1$ where types with $\sigma \leq \tilde{\sigma}$ will be announced good with certainty and the investor will invest. The CRA would be making positive expected profits then.

If $E(V_D|a = g) > 1$, then the investor will invest when the announcement is good. However, as Π_C is decreasing in e, the CRA can lower e slightly which still satisfies the above inequality. This will lead to higher profits to it. Thus, $E(V_D|a = g) > 1$, cannot be an equilibrium.

Finally, consider the case that $E(V_D|a = b) \geq 1$. As $E(V_D|a = g) \geq E(V_D|a = b)$, using the same arguments used in the case where $E(V_D|a = g) > 1$, suffices.

(b) We first show that e^* satisfies,

$$\int_0^{\bar{\sigma}} [V_D(\sigma) - 1]\alpha(\sigma, e^*) dF(\sigma) = 0.$$

We will then show that $e^* \in (0, 1)$.

Note that the CRA's profit decreases with e. Therefore, it will set the minimum e^* such that the investor breaks even. The expected profits to the investor is

$$\int_0^{\bar{\sigma}} [V_D(\sigma) - 1]\{\alpha(\sigma, e)[\gamma(e) - \beta(e)] - \beta(e)\} dF(\sigma).$$

From (a), we know that in equilibrium, $\gamma^* = 1; \beta^* = 0$. Let e^* be such that,

$$\int_0^{\bar{\sigma}} [V_D(\sigma) - 1]\alpha(\sigma, e^*) dF(\sigma) = 0.$$

If $e < e^*$, then from lemma 3, we have $H(\sigma, e) < 0$. Therefore, the investor will not invest if the announcement is good. Given lemma 2, it is evident that she will not invest if the announcement is bad. Thus, the CRA makes losses if $e < e^*$. Therefore, $e < e^*$ is not an equilibrium.

If $e > e^*, H(\sigma, e) > 0$. Therefore, the investor will invest when the announcement is good. However, as Π_c decreases in e, the CRA can strictly de better be setting e' where $e^* < e' < e$.

If $e^* = 0$, then $\alpha(p, 0) = \tilde{\sigma}$. Then either the investor does not invest in any types or invests in all types. If she does not, then no types will come to the CRA. This cannot be an equilibrium as the CRA can strictly do better by setting $e^* = 1$. If all types are invested in, then given A.2, $E(\sigma|a = g) = E(\sigma|a = b) < \tilde{\sigma}$. Therefore, $e^* = 0$ is not an equilibrium.

To argue that, $e^* < 1$ note that, with $e^* = 1$, $E(V_D|a = g) > 1$. We have argued in part (a) that this cannot be an equilibrium.

Finally, uniqueness follows from the fact that Π_C is decreasing in e. ■

- **Proof of Proposition 3.**

Proof: If the incentive payment has to be made *ex-ante*, it must be the case that, $\phi(.)$ is based only on the contractible variable, the announcement m. Denote $\Pi_c(m, e)$ as the expected profits to a CRA who sets e and announces that it has set m. Therefore, $\Pi_c(m, e) = X - c(e) + \phi(m)$. Note that, X is the fixed part which is paid by all those firms who wishes to get rated.

$\phi(m)$ will characterize a truthful revelation equilibrium iff,

$$\Pi_c(m, m) > \Pi_c(m, e) \qquad \text{and} \qquad \Pi_c(e, e) > \Pi_c(e, m).$$

It is easy to see that, with the incentive component only defined on $\phi(m)$, both the inequalities cannot be satisfied. ∎

- **Proof of Proposition 4.**

Proof: Let $e_1 > e_2$. We will show that, with Ω, $\Pi_c^{\Omega}(e_1, e_1) > \Pi_c^{\Omega}(e_1, e_2)$ is never possible. Note, from the expression of $\Pi_c^{\Omega}(m, e)$, the component $X - c(e)$ is decreasing in e. It remains to show therefore,

$$\theta_{\Omega}(e_1)t\int_0^{\bar{\sigma}} V_E(\sigma)[\gamma(e_1) - \beta(e_1)](\alpha(\sigma, e_1) - \alpha(\sigma, e_2)dF(\sigma) \leq 0.$$

Denote, $\alpha_j \equiv \alpha(\sigma, e_j); \gamma_j \equiv \gamma(e_j); \beta_j \equiv \beta(e_j); \theta_{\Omega j} \equiv \theta_{\Omega}(e_j); j = 1, 2$. Rewrite the LHS of the above as,

$$[\gamma_1 - \beta_1]\theta_{\Omega_1}t\left\{\int_0^{\tilde{\sigma}} V_E(\sigma)(\alpha_1 - \alpha_2)dF(\sigma) + \int_{\tilde{\sigma}}^{\bar{\sigma}} V_E(\sigma)(\alpha_1 - \alpha_2)dF(\sigma)\right\}.$$

From the properties of α it is evident that $\alpha_1 > \alpha_2$ for $\sigma < \tilde{\sigma}$ and $\alpha_1 \leq \alpha_2$ for $\sigma \geq \tilde{\sigma}$ and also $\alpha_1 - \alpha_2$ is linear in σ. Therefore, the first integral is positive while the second one is negative. Now, as $\tilde{\sigma} < \sigma^e$, the measure of types under the first integral is less than those under the second. Finally, with V_E increasing in σ, more weights are attached to the second integral implying that the sum of both the integrals are negative. ∎

- **Proof of Proposition 5.**

Proof: The proof is similar to that in proposition 4. We first prove that for $\tilde{\sigma} \to \sigma^e$, $\exists\theta^*_{\Delta}$ such that the inequalities in (A) are satisfied. This will complete the existence proof. Next we will show that, in a fully revealing equilibrium, it must be the case that, $\theta^*_D(m)$ has to be increasing in the announcements, m.

Note that, in equilibrium, $\forall e \in [0, 1], \gamma(e) = 1, \beta(e) = 0$.

Step I:

$$\forall\tilde{\sigma} \to 0, \quad \int_0^{\bar{\sigma}} V_D(\sigma)[\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma) < 0.$$

Rewrite the above as,

$$\int_0^{\tilde{\sigma}} V_D(\sigma)(\alpha_1 - \alpha_2)dF(\sigma) + \int_{\tilde{\sigma}}^{\bar{\sigma}} V_D(\sigma)(\alpha_1 - \alpha_2)dF(\sigma).$$

Now from the properties of α, we have $\alpha_1 > \alpha_2$ for $\sigma < \tilde{\sigma}$ and $\alpha_1 \leq \alpha_2$ for $\sigma \geq \tilde{\sigma}$. Therefore, the first integral is positive while the second is negative. With $\tilde{\sigma} \to 0$, sum of the integrals is negative.

Step II:

$$\forall \tilde{\sigma} \to \sigma^e, \quad \int_0^{\bar{\sigma}} V_D(\sigma)[\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma) > 0.$$

Rewrite the above as,

$$\int_0^{\tilde{\sigma}} V_D(\sigma)(\alpha_1 - \alpha_2)dF(\sigma) + \int_{\tilde{\sigma}}^{\bar{\sigma}} V_D(\sigma)(\alpha_1 - \alpha_2)dF(\sigma).$$

The steps are similar to Step I. Note that, as $\tilde{\sigma} \to \sigma^e$, the measure of types in the first integral is approximately equal to the measure of firms in the second. However, as $V_D(\sigma)$ is decreasing in σ, lesser weight is now attached to the negative component.

Steps I and II imply that for both the inequalities in (A) to be satisfied, it is necessary that $\tilde{\sigma}$ is not too low.

For sufficiently low marginal costs (i.e., low $|c(e_1) - c(e_2)|$), if $\theta_\Delta(e_1)$ is sufficiently large while $\theta_D e(e_2)$ is sufficiently small, both the inequalities will be satisfied.

Monotonicity:

Rewrite (A) as,

$$\begin{aligned} & \Pi_c^\Delta(e_1, e_1) > \Pi_c^\Delta(e_1, e_2) \quad \text{and} \quad \Pi_c^\Delta(e_2, e_2) > \Pi_c^\Delta(e_2, e_1) \Rightarrow \\ (A) \quad & t\theta_\Delta(e_1) \int_0^{\bar{\sigma}} V_D(\sigma)[\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma) \\ & \qquad > c(e_1) - c(e_2) \geq \\ & t\theta_\Delta(e_2) \int_0^{\bar{\sigma}} V_D(\sigma)[\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma). \end{aligned}$$

Note, for $\tilde{\sigma} \to \sigma^e$,

$$\int_0^{\bar{\sigma}} V_D(\sigma)[\alpha(\sigma, e_1) - \alpha(\sigma, e_2)]dF(\sigma) > 0.$$

Therefore, for the inequalities to hold it must be the case that $\theta^*_\Delta(e_1) > \theta^*_\Delta(e_2), \forall e_1 > e_2$. ■

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